Asymmetric Perception of Firm Entry and Exit

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Abstract

Many models of firm behavior in economics fail to conceptualize the correct process of firms' action or to reproduce the empirical macroeconomic outcome. The reason is that the behavior of firms in the economy does not follow a strict model that a representative agent can conceptualize. The Quantal Response Statistical Equilibrium framework has been developed by Ellis Scharfenaker and Duncan Foley to bridge the methodological gap between unobservable, individual behavior and the observed distribution of parameter at the macro level. The model is based on the entropy maximizing principle to ensure the largest degree of freedom. It uses the observed macroeconomic distribution of all firm's return rate as a constraint to the model. The macroeconomic outcome constrains the individual probabilistic behavior of firms. The decision-making process of firms does differ for entry and exit decisions. The separate threshold and behavioral temperature cannot be captured in most models. I provide a preliminary overview of how a Quantal Response Statistical Equilibrium framework could work if the entry and exit decisions are asymmetric in their threshold and behavioral temperature. Compared to the baseline Quantal Response Statistical Equilibrium framework or other classical competition models, the asymmetric Quantal Response Statistical Equilibrium model is more capable of capturing the data. The asymmetric Quantal Response Statistical Equilibrium model allows for a detailed analysis of how the entry and exit behavior is induenced and changed over time. The addition of asymmetric behavior creates a model closer to the economic reality. The model allows for seperate thresholds for firm entry and exit decisions of firms into a market. These separate thresholds are accompanied by different behavioral temperatures. The behavioral temperatures capture the reactivity of firms to divergence from the threshold. As the model captures the data-generating process of individual firms and the observed macroeconomic outcome, it provides a powerful tool for modeling, understanding, and evaluating firm behavior.

1 Competition and Equilibrium

In modern economics, competition is like the holy grail; it is the goal and origin of economic and political action, leading to prosperity and freedom. It became a black-box that describes vaguely a concept of interaction and the outcome of the economic system. But this understanding of competition is relatively new for economic thinkers. It originates within the last century and is a shift from the way how humans considered the economy throughout time.

One of the first people who approached economic questions was Thomas of Aquinas. His focus was not directly on competition itself, but rather the economic system as a natural order. The natural order in which we act and interact is given by God to human mankind. To change and adapt the system is against the intention of God, an affront against the almighty, heresy in its purest form (Medema & Samuels 2013, p. 18-34).

François Quesnay takes a similar approach, focusing on the natural circumstances of the economy. As a physician at the court in Versailles, his economic modeling was inspired by the medical progress and knowledge of his time. The Tableau Économique mimicked the blood flow in the human body, increasing blood pressure or releasing 'bad blood' leads to better economic outcomes. Market interventions are similar to the interventions of a physician into the human body.

More specific on the economic order was Jacob Viner. For him the purpose of the economic system was more important than the preservation of the system. As a mercantilist, he was concerned how to increase state power through economic activity. The existence of domestic monopolies was a necessity to create a favorable balance of trade, which leads to economic wealth and therefore more power. As power has been considered by the mercantilists as a relative matter, the ratio of wealth compared to other countries mattered more than the absolute level of wealth. The mercantilists subordinated all economic activity to the goal of power creation, the political agenda was clear on its priorities (Medema & Samuels 2013, p. 267-274).

Thomas Mun had a similar focus in his studies as Viner. He analyzed what form of economic activity determines and therefore increases the wealth of nations. Setting himself ideological apart from the mercantilists, he focused on the strength of the domestic markets and how the state is integrated into the world economy. The connection and interaction of states in foreign trade became for him the driver of strength, not the separation and protectionist approach taken by mercantilistic states (Medema & Samuels 2013, p. 35-44).

The argument by John Locke in favor of free trade was not a statement for market liberalizations as it is discussed today. It was an answer to the mercantilistic approach of the time. The domestic monopolies proposed by the mercantilists were a threat to the political power of merchants within the state. The shift of power from oligarchic merchants towards the state was threatening the political standing of the class, something opposed by Locke (Medema & Samuels 2013, p. 57).

The work of Adam Smiths must be seen in a similar light. Even if he argues in favor of free markets, his argument is driven by his objection to the mercantilistic system. It is the monopolist who keeps "the market constantly under-stocked, by never fully supplying the effectual demand, sell their commodities much above the natural price, and raise their endowments, whether they consist in wages or profit, greatly above their natural price." (Smith 1976, . 78). Smith differs between the natural price and the price that is actually charged. The natural price is the sum of wages paid to the farmer who produced the commodity and the profit to the landlord who provided the land. The price charged fluctuates around this level, depending if a commodity is 'over-stocked' or 'under-stocked'. It s worth noticing that the natural price concept of Smith is similar to the modern understanding of accounting profits. The profit of the landlord is already included into the price calculation. If a monopolist is bale to charge a price above the natural price, there is a second type of profit. To avoid confusion with the two forms of profit, I will use the Marxian terminology of 'rent' in the following to describe the profit which is created by the intentional under-stocking by the monopolist.

Smith was in favor for the widening of the market as it is "agreeable enough to the interest of the publick [sic!]" (Smith 1976, p. 267). To narrow the market creates benefits to only a few and is "an absurd tax upon the rest of their fellow citizens" (Smith 1976, p. 267) and must strictly be opposed. The argument is that the existence of monopolies and their creation of rent as hurtful to the economy. That the creation of rent stands opposed the virtue of frugality is another 'natural order' style argument in justifying a specific economic order.

A clearer understanding and description of the hurtfulness of monopolies is provided by Karl Marx. The tendency for the equalization of the rate of profit is a uncontested concept for Marxist and neoclassical economists. It is the necessary outcome from individual search for profitability as capital will move from sectors with a low rate of profit to sectors with a higher rate. Capital will always seek the highest returns and therefore tend to equalize the profit rates. The outflow of capital from sectors with a low return rate will reduce the pressure of competition and allows for an increase in output prices. This will lead to higher return rates. The opposite effect takes place in sectors with a high return rate, the inflow of capital will increase the pressure on sales and prices and therefore lead to lower rates of return. The free flow of capital is an important mechanism for the tendency to equalize the rate of return. Barriers to this, especially in the form of monopolization restrict this mechanism and allow for rents beyond the natural price (Marx & Korsc 2009, Buch 3).

A more distanced approach to the barriers of competition has been taken by the Austrian school in economics. In the Austrian school of thought, monopolies are not a problem as they will always be challenged by (potential) competitors. While the *arbitrageur* is able to generate excessive profits, it creates the incentive for other actors to enter the market (Hayek 1945). It is not common for the economic literature to argue that the classical authors like Smith and Marx are conceptually similar to the Austrian school. While there are significant differences in the understanding of persistence of monopoly and efficiency of market intervention, the core of competition and how it works are similar: excessive profits or rents lead to entry of firms and consecutive to a tendency of equalization of the rate of return.

Most economic models provide a solution in a fix point equilibrium. These form of equilibria is a specific allocation at a specific point in time, or a specific correlation between two variables. These fix points are provided with some uncertainty through the introduction of error terms into the equations. Solutions in complexity theory are different. Through the focus on algorithmic behavior of agents and probabilistic, rather than algebraic math, the nature of modeling is different. The solution of problems in such problems is an equation whose parameter differ over time (Arthur 2021).

Complexity theory does not contribute to the understanding of competition but rather provides a theoretical framework to model competition. Instead of concentrating on observable outcomes, complexity theory puts its focus on the actions within the system. Brian Arthur (2021) highlights the difference of complexity theory to other theories in economics by comparing the subject of analysis to nouns and verbs. While most economic modeling is concentrated on the correlation and connection between nouns (GDP, total assets, as examples), complexity theory highlights how those 'nouns' and object of study interact. With respect to competition this allows to focus on the flow of capital from low revenue to high revenue sectors. There are several ways to model this flow of capital. Most commonly are drift and diffusion models (Alfarano et al. 2012) or quantal response models that focus more on the individual choice of actors (Scharfenaker & Foley 2017, Scharfenaker 2020). The outcome of these kind of models is identical. Both lead to a statistical equilibrium of competition. A statistical equilibrium incorporates the multiple algorithms of firm's decision making and action to create an outcome which incorporates heterogeneous agents and individualism (Bouchaud 2009).

2 Characteristics of Statistical Equilibrium Competition

Two models dominate the literature to explain the distribution of the rate of profit. On one hand the Subbotin distribution, on the other hand the Quantal Response Statistical Equilibrium Model (QRSE). Both models have in common that they provide a good fit to the empirical distribution. They differ in the way how they explain the underlying data-generating process. The 'Subbotin-approach' relies on an external model, like a drift-diffusion approach (Alfarano et al. 2012). The QRSE approach does derive the theoretical distribution based on the data-generating behavior (Scharfenaker & Foley 2017). Both models imply a classic competition for the data-generating process.

2.1 Subbotin Distribution Modeling

The Subbotin distribution (also know as exponential power distribution) was introduced by M. Subbotin (1923). It is a three-parameter distribution which incorporates the Laplace and normal distribution as special cases. Bottazzi & Secchi (2006) introduce the symmetric Subbotin distribution (SSD) as a relevant model to analyze firm growth rates, later the distribution was adapted to explain the distribution of profit rates (Alfarano & Milaković 2008).

The realization of the Subbotin distribution for a competitive process is justified by Bottazzi & Secchi (2006) through "a permanent struggle within an extremely volatile environment" (p. 253). The number of random shocks that a firm can experience will determine the outcome of the observed distribution. The authors make the case that if the number of possible shocks is close to the number of firms, the randomness of the assignment will lead to a normal distribution. Once the number of possible shocks exceeds the number of firms significantly (in the case of the paper by factor 100), the observed outcome will be Laplacian. This is considered supportive evidence for an underlying positive feedback mechanism in the process of firm growth.

The idea of a Subbotin distribution with a focus on the Laplace distribution was picked up by Alfarano & Milaković (2008) to describe the behavior of profit rates in the US economy for surviving firms. Observing a similar distribution for the profit rate as to the growth rate of firms is explained with the close proximity of the driving forces in the process of classical competition.

The link to the concept of classical competition gets picked up four years later when the same authors provide a more technical explanation (Alfarano et al. 2012). The authors introduce a *drift-diffusion model* where a random term (in form of Wiener increments) and a systematic effect of drift term model the complexity of a competitive environment. The systematic drift is the tendency to equalize the rate of profit which affects all firms. The idiosyncratic shocks in the diffusion part address the changes of taste and technology. The authors conclude that the Laplace distribution provides an extraordinary fit to the rate of profit for firms with a long lifespan.

The Laplace distribution provides a great fit for a selection of firms that are neither entering nor exiting the sample. If the sample includes the mobility into and out of existence, the model reaches its limitations. The entry-exit process, especially over the last few decades does not follow a symmetric pattern. The asymmetry of the process can be captured by an asymmetric Subbotin distribution (ASD) (Mundt & Oh 2019). The asymmetric Subbotin distribution has five parameters, a mode and two shape and scale parameter, respectively, that describe each tail (Bottazzi & Secchi 2011). The separate description of the tails allows the model to incorporate structural differences for firms in each tail. This can be linked to the qualitative differences due to entry and exit dynamics (Mundt & Oh 2019).

Both distributions, the SSD and ASD, can be derived by maximizing the entropy of the two conditional moment constraints. This means that the distribution under observation is the least biased distribution for the given constraints (Mundt & Oh 2019). The SSD is defined by the location parameter m, the positive scale parameter a, and the positive shape parameter b:

$$f_{SSD}\left(x;b,a,m\right) = \frac{1}{2ab^{\frac{1}{b}}\Gamma\left(\frac{1}{b}+1\right)} \exp^{-\frac{1}{b}\left|\frac{x-m}{a}\right|^{b}}$$

In the case of the ASD, the shape and scale parameter are split into the left and the right tail, indicated by the index l and r:

$$f_{ASD}\left(x;b_l,b_r,a_l,a_r,m\right) = \frac{1}{a_l b_l^{\frac{1}{b_l}} \Gamma\left(1+\frac{1}{b_l}\right) + a_r b_r^{\frac{1}{b_r}} \Gamma\left(1+\frac{1}{b_r}\right)} \times \begin{cases} \exp\left(-\frac{1}{b_l} \mid \frac{x-m}{a_l} \mid^{b_l}\right) & x < m \\ \exp\left(-\frac{1}{b_r} \mid \frac{x-m}{a_r} \mid^{b_r}\right) & x \geq m \end{cases}$$

2.2 Quantal Response Statistical Equilibrium Modeling

The Quantal Response Statistical Equilibrium framework (QRSE) as proven to be an extremely powerful and accurate instrument to analyze non-observable action parameters. The framework makes it possible to disaggregate the observed macroeconomic distribution of a variable. Instead of analyzing the occurred actions, the model allows to determine the action probability. Researchers were able to analyze the relevant actions in the housing and stock market (Ömer 2018, Citera 2021), as well as induced technological change (Yang 2018).

Existing models analyze the actions *buy* and *sell* in different terminology. What these models have in common is the fact that the action variable determines the counteraction as well. This means that *buying* is considered equal to *not-selling*. This model adds a third and a fourth dimension to the toolkit of a firm. Rather than equalizing *buying* with *non-selling*, it adds the option of *holding* and *non-holding*. The classic symmetric QRSE model is extended into an asymmetric QRSE model (aQRSE).

It can be argued in the context of stock trading that *holding* is the same as the combination of *selling* and *buying* (with the assumption of the absence of transaction costs). The same analogy can be made for physical assets of a firm in case of *enter* or *exit* of a market. Allowing for the additional two action *stay-out* and *stay-in* does not pool firms into the *enter* or *exit* decision.

To introduce a non-action component into the analysis is not just an improvement in theoretical accuracy, but also allows for a more detailed analysis of the underlying problem. We are now able to analyze the *enter* and *exit* actions separately rather than as the inverse probability from each other. This allows a researcher to analyze the two actions separately. We can therefore have asymmetric *enter* and *exit* distributions. They both provide different thresholds μ and different behavioral temperatures.

The two additional parameters for the model allow for a separate analysis of the *enter* and *exit* variable. Both actions are now independent of each other as they are in the decision making of firms. It is realistic that a firm sees its *enter*-threshold different from its *exit*-threshold. It can be argued that such a different threshold comes from the different valuation of change. One can frame this as the stickiness of the current status of a firm. A firm which is already in a market is also more likely to accept a lower return on its investment than a firm would require to actively enter the market. The economic arguments for this can be either the ‘sunk-cost fallacy’ , that the effort of entering a market is not appropriately priced in, or that a different risk-aversion exist. Most likely is a behavior that combines all three aspects. On one hand, a firm which is in the market values the risk of being left out higher than the risk of remaining in the market and risking its assets due to managerial misjudgment. On the other hand, a firm outside will require a higher risk premia to enter a market and with that the economic risk, to offset the (safer) investment into other assets.

The behavioral temperature T will also be different for firms depending on their status towards the market. While we justify the different thresholds based on individual preferences and traits, the difference in the behavioral temperature might be of a more macro-economic nature. The temperature determines how fast a firm reacts to the difference between its own realization x and the threshold μ.

Derivation of the Asymmetric Case

The aQRSE model allows us to disagregate observed outcomes into actions. Those actions cannot and are not observed. In this case we concentrate on ROIC as the observed outcome and the action to enter or exit a market. The actions *exit* and *enter* are not complementary. Those two different actions allow for four different decisions: *exit*, *stay-in*, *enter*, and *stay-out*. The goal is to derive a probability for a specific action, given the observed outcome. We will make use of the conditional probability theorem:

$$P\left[action, x\right] = P\left[action|x\right] P\left[x\right]$$
$$P\left[action|x\right] = \frac{1}{\exp^{\frac{x-\mu}{T}} + 1}$$

The idea is that the marginal frequency of action depends conditional on the outcome. The quantal response function serves as the reaction function. It takes into account how far an agent diverged from the threshold μ and the behavioral temperature T. The traditional QRSE argues that *enter* and *exit* are exclusive options where the probability *enter* is 1 less the frequency of *exit* (Scharfenaker & Foley 2017). Here we argue that both action are independent. The idea that entry and exit are independent actions which are driven by different managerial reasoning (Scharfenaker & Foley 2017). The separation of action requires for both actions to have separate threshold parameter μ and separate behavioral temperatures T. We can now specify the conditional frequencies of action conditional on outcome. The index α and β indicate if the threshold μ and behavioral temperature T is addressing *enter* (α) or *exit* (β).

$$P\left[enter|x\right] = \frac{1}{\exp^{-\frac{x-\mu_\alpha}{T_\alpha}} + 1}$$
$$P\left[exit|x\right] = \frac{1}{\exp^{\frac{x-\mu_\beta}{T_\beta}} + 1}$$

We can use the conditional frequencies to create the joint frequency matrix (Table 1).

We can now derive the joint frequency matrix for any x:

To separate *enter* and *exit* adds a third and fourth action to the toolkit of a firm. The action *stay-out* is fundamentally different from *exit*. In the baseline QRSE, a firm

Table 1: The joint frequency matrix of the aQRSE model.

	enter	*exit*	
exit	$\frac{1}{\exp^{-\frac{x-\mu_\alpha}{T_\alpha}}+1}\frac{1}{\exp^{\frac{x-\mu_\beta}{T_\beta}}+1}$	$\frac{1}{\exp^{\frac{x-\mu_\alpha}{T_\alpha}}+1}\frac{1}{\exp^{\frac{x-\mu_\beta}{T_\beta}}+1}$	$\frac{1}{\exp^{\frac{x-\mu_\beta}{T_\beta}}+1}$
enter	$\frac{1}{\exp^{-\frac{x-\mu_\alpha}{T_\alpha}}+1}\frac{1}{\exp^{-\frac{x-\mu_\beta}{T_\beta}}+1}$	$\frac{1}{\exp^{\frac{x-\mu_\alpha}{T_\alpha}}+1}\frac{1}{\exp^{-\frac{x-\mu_\beta}{T_\beta}}+1}$	$\frac{1}{\exp^{-\frac{x-\mu_\beta}{T_\beta}}+1}$
Marginal Frequency	$\frac{1}{\exp^{-\frac{x-\mu_\alpha}{T_\alpha}}+1}$	$\frac{1}{\exp^{\frac{x-\mu_\alpha}{T_\alpha}}+1}$	1

which is outside of the market would have to *enter* and *exit* to remain in that position. In the aQRSE framework, it can chose the action *stay-out*. While the impact of the outcome of those two options is identical (in this case zero), it is a technical improvement which allows for the independent analysis of *enter* and *exit*. This argument applies for the *stay-in* action as well.

The no-impact action probability is constraint to a positive finite value δ which can be interpreted as a point of attraction between *enter* and *exit*. It acts as a feedback for the actions *enter* and *exit* and will later be specified as the average of the two thresholds, weighted by their relative behavioral temperature.

$$\int f\left(enter\right) E\left[x|enter\right] dx \leq \delta \leq \int f\left(exit\right) E\left[x|exit\right] dx$$

The difference between the *enter* and *exit* must be larger than zero, if the distance of x to the point of attraction δ is incorporated.

$$\begin{aligned} &\int f\left(enter\right) E\left[x|enter\right]\left(x-\delta\right) dx - \int f\left(exit\right) E\left[x|exit\right]\left(x-\delta\right) dx \\ =&\int \left(f\left[enter|x\right] f\left[x\right]\left(x-\delta\right) - f\left[exit|x\right] f\left[x\right]\left(x-\delta\right)\right)\ dx \\ =&\int \left(\frac{1}{\exp^{-\frac{x-\mu_\alpha}{T_\alpha}}+1} - \frac{1}{\exp^{\frac{x-\mu_\beta}{T_\beta}}+1}\right) f\left[x\right]\left(x-\delta\right)\ dx \leq 0 \end{aligned}$$

We can make use of the fact that the exponential part $\exp\left(x\right)$ can be rewritten as the fraction of two hyperbolic tangent functions $\frac{1+\tanh\left(\frac{x}{2}\right)}{1-\tanh\left(\frac{x}{2}\right)}$. We can therefore rewrite the conditional *enter* and *exit* as follows:

$$f\left[enter|x\right]=\frac{1}{\exp^{-\frac{x-\mu_{\alpha}}{T_{\alpha}}+1}}$$
$$=\frac{1}{2}\left(1-\tanh\left(-\frac{x-\mu_{\alpha}}{2T_{\alpha}}\right)\right)$$

$$f\left[exit|x\right]=\frac{1}{\exp^{\frac{x-\mu_{\beta}}{T_{\beta}}+1}}$$
$$=\frac{1}{2}\left(1-\tanh\left(-\frac{x-\mu_{\beta}}{2T_{\beta}}\right)\right)$$

The no-impact action probability can be rewritten as follows:

$$\int f\left(enter\right)E\left[x|enter\right]dx-\int f\left(exit\right)E\left[x|exit\right]dx$$
$$=\int\left(\frac{1}{2}\left(1-\tanh\left(-\frac{x-\mu_{\alpha}}{2T_{\alpha}}\right)\right)-\frac{1}{2}\left(1-\tanh\left(-\frac{x-\mu_{\beta}}{2T_{\beta}}\right)\right)\right)f\left[x\right]\left(x-\delta\right)\ dx$$
$$=\int\left(\frac{1}{2}\left(\tanh\left(-\frac{x-\mu_{\beta}}{2T_{\beta}}\right)-\tanh\left(-\frac{x-\mu_{\alpha}}{2T_{\alpha}}\right)\right)\right)f\left[x\right]\left(x-\delta\right)\ dx$$

As we impose maximal uncertainty for the model, we will maximize the entropy of the model. We can decompose this joint entropy of the marginal frequencies $f\left[x\right]$ into two components, the entropy of the marginal distribution and the average entropy of the conditional distribution. We will write the binary entropy function of the two actions for simplicity as R:

$$H=-\int f\left[x\right]\log\left(f\left[x\right]\right)\ dx+\int f\left[x\right]R\ dx,$$

where

$$R=-\sum_{a}f\left[a|x\right]log\left[f\left[a|x\right]\right]$$
$$=-\left(\frac{1}{\exp^{-\frac{x-\mu_{\alpha}}{T_{\alpha}}+1}}\log\left(\frac{1}{\exp^{-\frac{x-\mu_{\alpha}}{T_{\alpha}}+1}}\right)+\frac{1}{\exp^{\frac{x-\mu_{\beta}}{T_{\beta}}+1}}\log\left(\frac{1}{\exp^{\frac{x-\mu_{\beta}}{T_{\beta}}+1}}\right)\right)$$

We want to widen the difference between the weighted conditional outcome expectations to clearly distinguish the *entry* and *exit* action. By applying a maximum entropy program, we ensure that the parameter we estimate are as uniformed distributed as possible. By giving the conditional action frequencies as constraints into the program we can ensure the least-informed (and therefore the widest) distribution of the parameter.

The Maximum entropy program is defined as following:

$$\max H = -\int f[x]\log\left(f[x]\right)\,dx + \int f[x]\,R\,dx$$

$$\text{subject to } \int f[x]\,dx = 1$$

$$\int\left(\frac{1}{2}\left(\tanh\left(-\frac{x-\mu_\beta}{2T_\beta}\right) - \tanh\left(-\frac{x-\mu_\alpha}{2T_\alpha}\right)\right)\right)(x-\delta)\,dx \le \delta$$

We can translate the maximum entropy problem into a Lagrangian optimization problem with the variables λ and σ:

$$\begin{aligned}\mathcal{L}\left[f[x],\lambda,\gamma,\sigma\right] &= -\int f[x]\log\left(f[x]\right)\,dx + \int f[x]\,R\,dx \\ &\quad -\lambda\left(\int f[x]\,dx - 1\right) \\ &\quad -\sigma\left(\int\left(\frac{1}{2}\left(\tanh\left(-\frac{x-\mu_\beta}{2T_\beta}\right) - \tanh\left(-\frac{x-\mu_\alpha}{2T_\alpha}\right)\right)\right)(x-\delta)\,dx - \delta\right)\end{aligned}$$

We can solve the first-order condition to get the maximum entropy marginal frequencies

$$\hat{f}[x] = \exp\left(-\left(1+\lambda\right)\right)\exp\left(R\right)\exp\left(-\sigma\frac{1}{2}\left(\tanh\left(-\frac{x-\mu_\beta}{2T_\beta}\right) - \tanh\left(-\frac{x-\mu_\alpha}{2T_\alpha}\right)\right)(x-\delta)\right)$$

and the marginal distribution

$$\hat{f}[x] = \frac{\exp\left(R\right)\exp\left(-\sigma\frac{1}{2}\left(\tanh\left(-\frac{x-\mu_\beta}{2T_\beta}\right) - \tanh\left(-\frac{x-\mu_\alpha}{2T_\alpha}\right)\right)(x-\delta)\right)}{\int\exp\left(R\right)\exp\left(-\sigma\frac{1}{2}\left(\tanh\left(-\frac{x-\mu_\beta}{2T_\beta}\right) - \tanh\left(-\frac{x-\mu_\alpha}{2T_\alpha}\right)\right)(x-\delta)\right)\,dx}$$

This allows us to derive the conditional outcome frequencies:

$$f\left[x|enter\right] = \frac{f\left[enter,x\right]}{f\left[enter\right]}$$
$$= \frac{\frac{1}{\exp^{-\frac{x-\mu_\alpha}{T_\alpha}}+1} f\left[x\right]}{\int \frac{1}{\exp^{-\frac{x-\mu_\alpha}{T_\alpha}}+1} f\left[x\right]\ dx}$$
$$= \frac{\frac{1}{\exp^{-\frac{x-\mu_\alpha}{T_\alpha}}+1} \exp\left(R - \frac{\sigma}{2}\left(\tanh\left(-\frac{x-\mu_\beta}{2T_\beta}\right) - \tanh\left(-\frac{x-\mu_\alpha}{2T_\alpha}\right)\right)(x-\delta)\right)}{\int \frac{1}{\exp^{-\frac{x-\mu_\alpha}{T_\alpha}}+1} \exp\left(R - \frac{\sigma}{2}\left(\tanh\left(-\frac{x-\mu_\beta}{2T_\beta}\right) - \tanh\left(-\frac{x-\mu_\alpha}{2T_\alpha}\right)\right)(x-\delta)\right)\ dx}$$

$$f\left[x|exit\right] = \frac{f\left[exit,x\right]}{f\left[exit\right]}$$
$$= \frac{\frac{1}{\exp^{\frac{x-\mu_\beta}{T_\beta}}+1} f\left[x\right]}{\int \frac{1}{\exp^{\frac{x-\mu_\beta}{T_\beta}}+1} f\left[x\right]\ dx}$$
$$= \frac{\frac{1}{\exp^{\frac{x-\mu_\beta}{T_\beta}}+1} \exp\left(R - \frac{\sigma}{2}\left(\tanh\left(-\frac{x-\mu_\beta}{2T_\beta}\right) - \tanh\left(-\frac{x-\mu_\alpha}{2T_\alpha}\right)\right)(x-\delta)\right)}{\int \frac{1}{\exp^{\frac{x-\mu_\beta}{T_\beta}}+1} \exp\left(R - \frac{\sigma}{2}\left(\tanh\left(-\frac{x-\mu_\beta}{2T_\beta}\right) - \tanh\left(-\frac{x-\mu_\alpha}{2T_\alpha}\right)\right)(x-\delta)\right)\ dx}$$

In order to achieve a strong point of attraction to which firms tend to convergence through their *enter* and *exit*, this point must be between the *enter* and *exit* thresholds. This threshold δ is the sum of the two thresholds, weighted with their relative behavioral temperature:

$$\delta = \mu_\alpha \frac{T_\alpha}{T_\alpha + T_\beta} + \mu_\beta \frac{T_\beta}{T_\alpha + T_\beta} \tag{1}$$

The probability density function (PDF) of the model is used to estimate the four parameter, from which the different distributions of *enter*, *exit*, *no impact*, and the PDF itself can be visualized.

This model is consistent with the symmetric baseline model (Scharfenaker & Foley 2017). In the symmetric baseline model QRSE the parameters μ_α and μ_β, as well as T_α and T_β must be identical. The aQRSE will collapse from six parameters to four parameters. The baseline QRSE is a simplified version of the here discussed aQRSE as it implies a single threshold for the *entry* and *exit* decision of firms.

3 Discussion of Parameter

The QRSE has three different kinds of parameter types. The parameter σ is determining the distribution's spread as a standard derivation, with large values indicating a wide dispersion of the firm performance.

The location parameter μ is a threshold that capitalists consider for their decision. When a capitalist has to choose between two actions, the capitalist is indifferent between the options they can choose from. In the discussion below, I will discuss the implication and impact of one and two different thresholds in the aQRSE model.

The temperature parameter T indicates how fast capitalists react to the divergence from the threshold. The lower their reaction temperature, the faster they react to any divergence from the threshold. The likelihood of acting in a specific way increases with the distance between the capitalist's realization and the threshold if the behavioral temperature is low. A high temperature means the option to choose between the two actions is available for values further away from the threshold.

The parameter δ has been endogenized in the process of deriving the final model and is therefore dependent on μ_α, μ_β, T_α, and T_β.

3.1 Discussion of the Threshold Parameter μ

The parameter μ is the threshold that capitalists consider normal. In the case of the two-option standard QRSE model the capitalist is indifferent between *enter* and *exit* at the threshold. Figure 1 shows that at the threshold $\mu = 0.1$, the capitalist will choose *no action* with a probability of 50%, and the action *enter* and *exit* with 25% respectively. The higher the capitalist's realization is compared to the threshold, the more likely they are to *enter*, while the probability of *exit* or *no action* decreases.

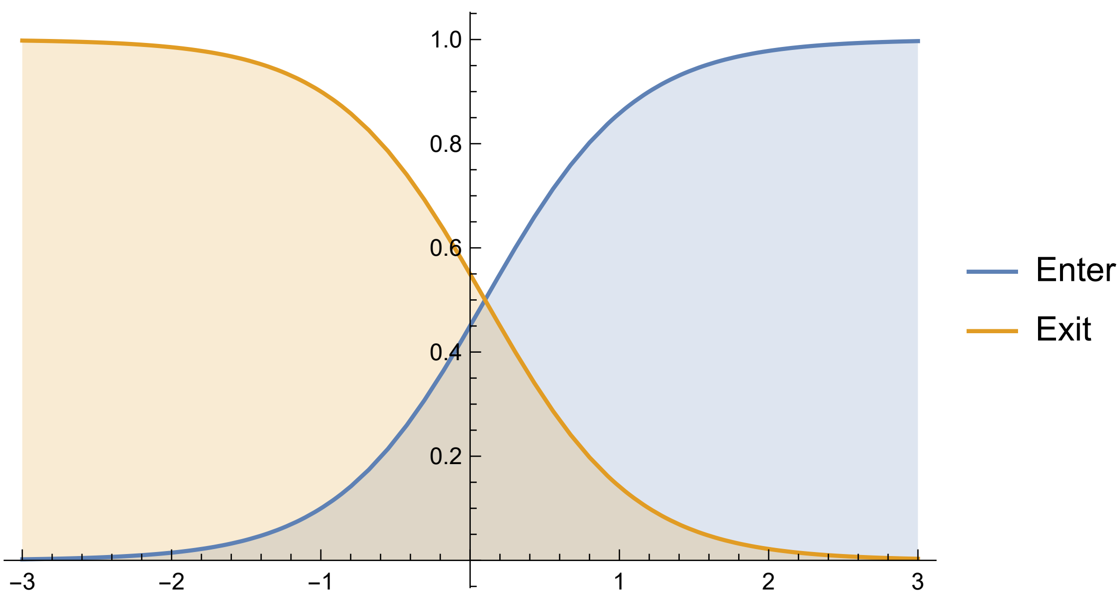

Figure 1: The threshold is $\mu = 0.1$ with the behavioral temperature $T = 0.5$. The Graph indicates the probability of a capitalist to chose a specific action depending on their own realization of the variable under consideration.

From an economic perspective, one threshold implies that a capitalist considers the threshold as the relevant metric for their decision disregarding the action. This implies that there is a desirable rate of return a capitalist considers as relevant concerning their decisions. Even the contrary decisions of *enter* and *exit* are bound to the same threshold. There needs to be an understanding that the decisions are different to the firm.

This is something a second threshold will capture better. In that case, the decisions *enter* and *exit* are fundamentally different from the perspective of the reference point for the capitalist. The *enter*/*exit* decisions are spread out further, creating a space between these two options in which the possibility of *no action* becomes the most likely (see Figure 2).

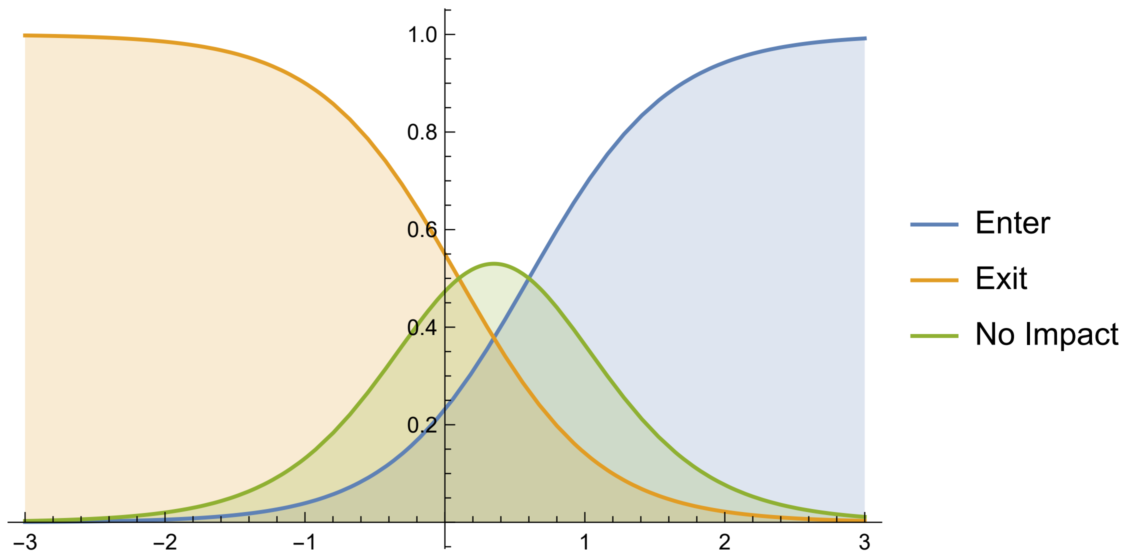

Figure 2: The threshold is $\mu_\alpha = 0.6$ and $\mu_\beta = 0.1$ with the behavioral temperature $T = 0.5$. The Graph indicates the probability of a capitalist to chose a specific action depending on their own realization of the variable under consideration.

While the action curves remain unaffected, the *no impact* option gains more weight. This implies that overall the number of actions of firms reduces. The action *enter* and *exit* become less likely overall as the mode of the performance indicator's PDF is between (or close by) the thresholds. The action is, therefore, more likely to happen in the tail. The behavior of firms becomes more stable by adding an additional threshold.

Adding a second threshold is a logical extension of the model from an economic perspective. It includes the fact that the decision to *enter* and *exit* is the result of different assessments by the capitalist. The single threshold implies that the *enter* and *exit* decisions are converse rather than independent. From an economic perspective, it is hard to imagine a scenario in which a capitalist decides to invest or divest based on their current performance. A capitalist is more likely to face the decision to *enter* than *no action* and similar at the other end of the spectrum.

The observation that the decision to act will happen in the tail and the majority of firms remain in what they are doing (*no impact*) is consistent with the findings that capitalists continue in their strategy without making adjustments. remain doing what they have done in the past because it worked (Hall & Hitch 1939). The closer the two thresholds are, the more volatile the decision-making of capitalists.

3.2 Discussion of the Threshold Parameter T

While the threshold μ determines a capitalist's decision makes, the temperature Parameter T indicates how reactive a capitalist is regarding their divergence from the threshold. The lower the behavioral temperature is, the clearer the decision is for a capitalist if they diverge from the threshold. A behavioral temperature of zero (or close to) creates a step function. The different temperatures for the *enter* and *exit* decision can create an asymmetric *no impact* distribution (see Figure 3).

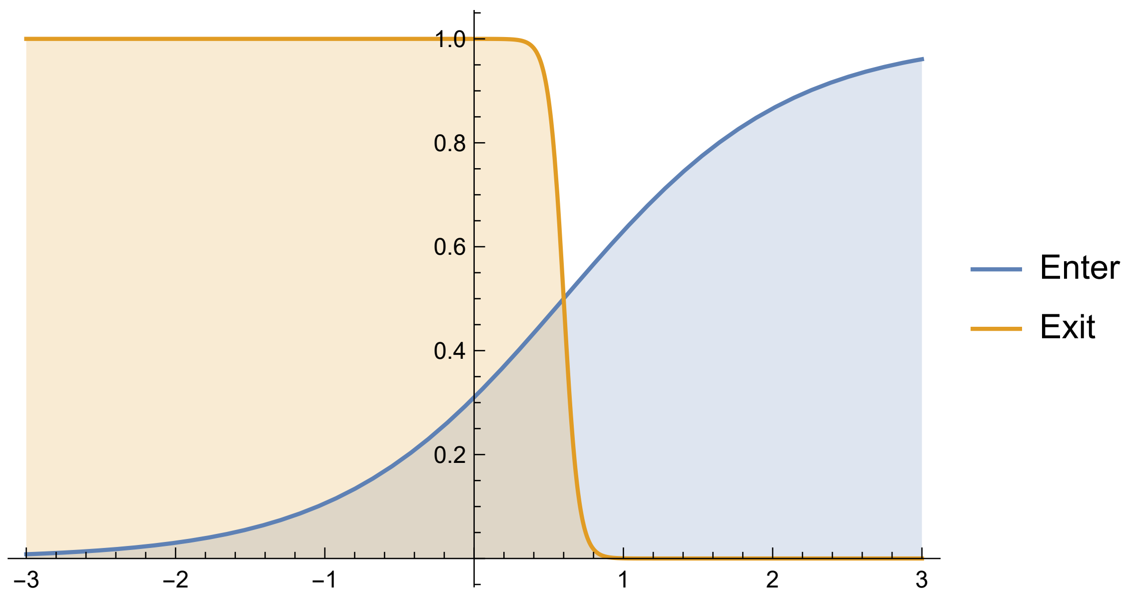

Figure 3: The threshold is $\mu_\alpha = 0.6$ and $\mu_\beta = 0.1$ with two separate behavioral temperatures, $T_\alpha = 0.05$ and $T_\beta = 0.75$. The Graph indicates the probability of a capitalist to chose a specific action depending on their own realization of the variable under consideration. The two different thresholds are chosen for a clearer visualization and do not impact the steepness of the reaction function.

The two different temperatures are economically reasonable. A capitalist is considering divergence from a threshold different depending on the nature of the decision. This may have economic as well as psychological reasons. The *no action*/*exit* decision may be driven by the idea of cutting losses as remaining continues to cost resources. In contrast, the *no action*/*enter* decision is driven by prospective profit.

A low *exit* temperature means capitalists have a threshold below which any action is undesirable. Once a firm hit this threshold, a capitalist will pull the plug. A high *exit* temperature indicates a reluctance to divest. The sunk cost fallacy and optimism may drive this into future developments. On the other end of the spectrum, a high *enter* means that a capitalist is reluctant to enter. The reason for that can be manifold, most likely the fear of commitment to the new investment. Such an investment is costly and consists of uncertainty. A low *enter* temperature, however, means that a specific threshold is like a gold standard, where *enter* becomes the logical response.

The opportunity to consider two different temperatures is a considerate addition to the model. It allows for a more detailed analysis of the *enter* and *exit* behavior. Creating independence of these two options is a step towards economic reality and the ability to analyze the similarities and differences in *enter* and *exit* behavior.

4 Specific Features of Additional Parameter

The traditional QRSE has been discussed in excessive detail by Scharfenaker (2022). The traditional model consists of one single threshold with one behavioral temperature. In this case, the model can reproduce the characteristics of several distributions like the Subbotin, normal, or Laplace distribution (Scharfenaker 2022, p. 245). With a behavioral temperature of zero, this model can imply a Dirac delta function at the point of the respective threshold. In this case, the market forces will instantly correct any derivation from the threshold and drive firms back towards the threshold. In such a case, we can speak of a perfect competition model in which all adjustments are instantaneous.

If the model allows for two different temperatures but only one threshold, the model can capture asymmetries in the data. The possibility of capturing asymmetry is so well-developed that the model can capture a bimodality (see Figure 4). Such a model can collapse to the symmetric base model if the behavioral temperature for *enter* and *exit* are identical.

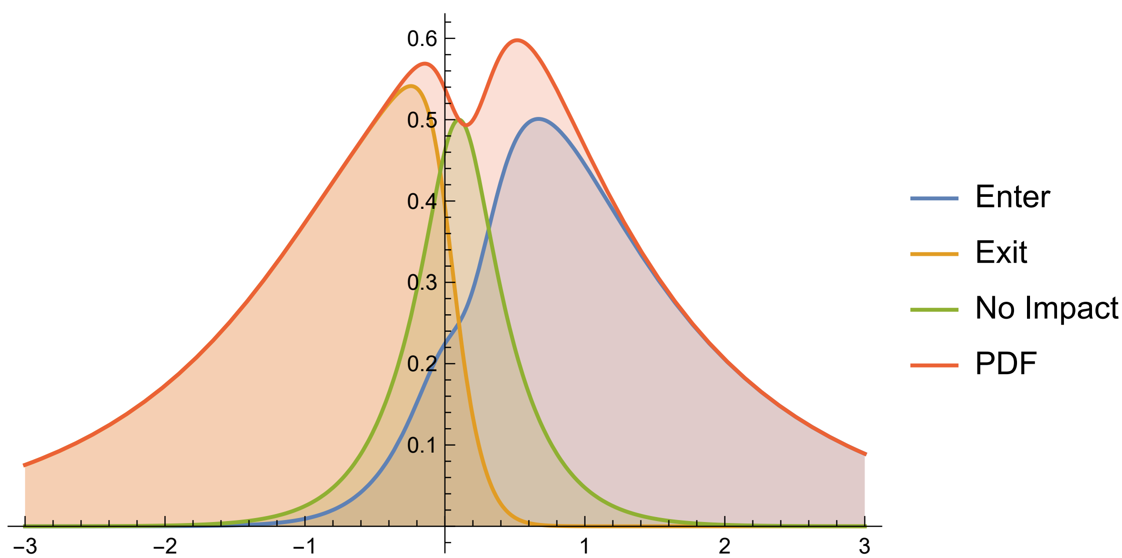

Figure 4: For the threshold is $\mu = 0.13$, the behavioral temperatures, $T_\alpha = 0.3$ and $T_\beta = 0.1$, and the standard deviation of $\sigma = 1.6$, the model expresses a bimodality in the PDF.

Even as bimodality cannot be observed in the data, a model which allows for such a behavior is important. As the micro behavior may lead to the bimodality, there must be additional forces in the competitive process which prevents such a behavior in reality. The clear single modality in rah of return measurements seems to be obtained by a convergence measure which is currently not incorporated into the model.

5 Discussion and Summary

Economic competition can be described and modeled in several different ways. While many models and approaches focus on reproducing the observed outcome of competition, they fail to analyze the underlying process leading to such a result. The value of any result must be questioned when the process and procedure that the result is obtained (Morishima 1984). A model which contradicts the process that is observed in reality but generates the correct outcome is like a blind chicken finding food.

The QRSE framework does not fall into the trap of sacrificing micro-economic behavior in order to achieve macro-economic outcomes. It rather favors the opposite by ensuring that the macro outcome emerges from the micro foundation. The detailed description of action parameters ensures that the model of competition captures the right process and leads to the observed macro-economic outcome. This feature sets the QRSE framework apart from other models of competition and competitive processes.

The QRSE framework with more threshold and temperature parameters than the baseline model provides additional insight into the micro behavior. It allows for a wide range of possible macro-economic pattern that can be reproduced by the model. Those features are the Subbotin distribution with its special cases of a normal or Laplace distribution, but also a Dirac delta function or bimodal distributions. By this, the QRSE framework can be applied to a large range of economic problems that have a clear economic micro foundation.

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